Technical Notes

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Thermal Curvature of Satellite Booms

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Nomenclature

dr, ds	= incremental arc length
dx	= incremental distance along base tangent direction
EI	= flexural rigidity
$\boldsymbol{k}$	= thermal curvature vector
ℓ	= boom length
$\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{p}''$	= position, tangent, and curvature vectors, respectively
q	= Lagrangian generalized coordinate
r	= arc length measured from base of boom
$\boldsymbol{t}$	= tangent vector
T	= kinetic energy
$\boldsymbol{T}$	= diametric temperature difference vector
V_E, V_S, V_t	= potential energy components associated with strain
w	= local deflection
β	= equivalent to $-\sigma_y$
θ	= derivative of local deflection with respect to arc length
σ_x, σ_y	= components of $\boldsymbol{\sigma}$ along and normal to base tangent, respectively
$\boldsymbol{\sigma}$	= unit sun line vector
ϕ	= angle between sun line and base tangent

Introduction

THERMAL deformation of a satellite boom has been characterized as induced curvature and as the effect of a stress function. These alternate interpretations have been noted previously,[1-4] and elastic dynamics were expressed in terms of departures from an equilibrium state.[5,6] These latter references involved the Radio Astronomy Explorer (RAE), which represents a "real world" successful realization of stable operation with nonrigid booms.

Uneven heating effects are complicated by a cycle of dependence, whereby thermal curvature is influenced by incident area which, in turn, is influenced by the curvature. Several additional factors may increase further the complexity of static and dynamic analysis; these factors include variations in cross section, absorptivity, or conductivity along boom length, superimposed load forces, tip masses, accompanying elongation or torsional deformations, rotations coupled to elastic modes, and damping. When these factors are missing,† however, there is a previously unknown closed-form solution for static slope at any distance along a fixed arc length with a single heat source. This solution is derived, followed by suggestions for further application.

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†It is not necessary to assume that torsion, elongation, etc., are absent, but that they are independent of the flexure which is the main subject under consideration here.

Of greater importance than the closed-form solution itself is the added modeling precision offered by the underlying formulation. Two characteristics of the present approach, often ignored in conventional analysis, are 1) positive definiteness for net elastic energy remaining after thermal deformations are taken into account, and 2) reduction of the axial coordinate in the presence of deflections, such that bending cannot cause changes in total arc length. Both of these features enhance model stability. References 5 and 6 employed this approach, but thermal delays were insignificant in that application. Although thermal lag is out of the present scope, a brief speculation is made in that regard after presentation of the analysis.

Mathematical Development

The analysis can begin by highlighting the basic approach to energy formulation with thermal effects. Consider an isolated homogeneous elastic cantilevered boom, whose cross section, absorptivity, and conductivity are uniform throughout its length, in the absence of load forces, elongation, and torsion. With $\boldsymbol{p}''$ and $\boldsymbol{k}$ used to denote total and thermally induced curvature, respectively, a positive definite net elastic strain energy V_E is proposed here, of the form

$$V_E = \frac{1}{2} EI \int_0^\ell (\boldsymbol{p}'' - \boldsymbol{k})^2 \mathrm{d}r \tag{1}$$

for usage in the Lagrange equation,

$$(\mathrm{d}/\mathrm{d}t)(\delta T/\delta \dot{q}) - \delta(T - V_E)/\delta q = 0 \tag{2}$$

Illustration of the vector form for curvature in Eq. (1) is facilitated by Fig. 1. Conventions are chosen so that the cantilever base tangent is along the positive x axis, and a deflection produced by thermal effects would be positive along the y axis; hence the y component of the unit sun line $\boldsymbol{\sigma}$ is always negative ($\sigma_y = -\beta$). It is also seen that

$$\sigma_x = \begin{cases} +\sqrt{1-\beta^2} & \text{(Fig. 1a)} \\ -\sqrt{1-\beta^2} & \text{(Fig. 1b)} \end{cases} \tag{3}$$

and, with the generalized coordinate in Eq. (2) expressed in terms of a local‡ deflection w, the vector to an arbitrary point at arc length r from the base of the boom is

$$\boldsymbol{p} = \begin{bmatrix} \int_0^r \sqrt{1-(\mathrm{d}w/\mathrm{d}s)^2}\,\mathrm{d}s \\ w \\ 0 \end{bmatrix} \tag{4}$$

which corresponds to a unit tangent vector,

$$\boldsymbol{t} \stackrel{\Delta}{=} \boldsymbol{p}' = \begin{bmatrix} \sqrt{1-(w')^2} \\ w' \\ 0 \end{bmatrix} \tag{5}$$

‡Quite commonly the local deflection is expressed as a product of (tip deflection)×(cantilever mode shape function), or as a sum of these products. That procedure is not necessary, however, to establish the points under discussion here.

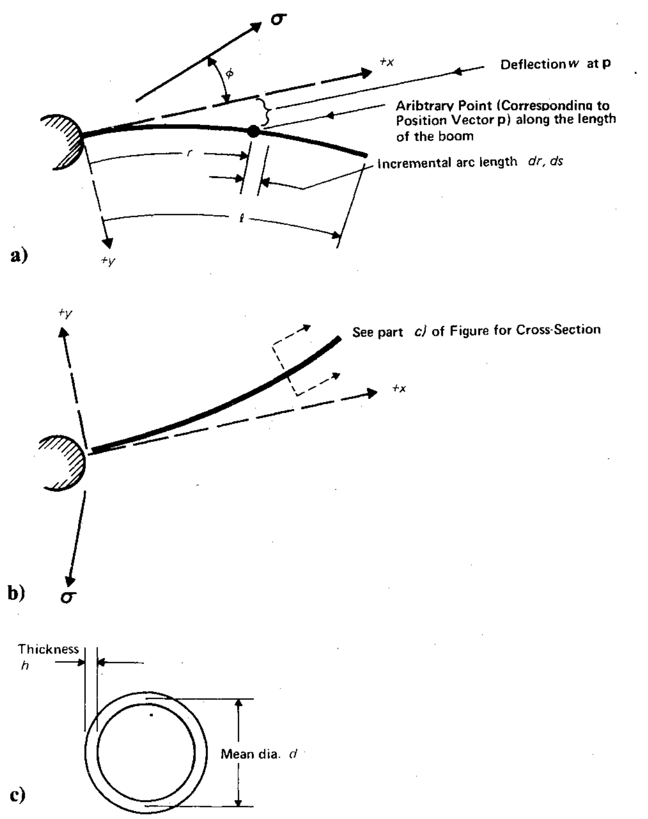

Fig. 1 Deflection geometry: a) $\phi < 90$ deg., Eq. (14); b) $\phi > 90$ deg, Eq. (14); c) enlarged cross section of boom.

and a curvature vector

$$p'' = \begin{bmatrix} -w'w''/\sqrt{1-(w')^2} \\ w'' \\ 0 \end{bmatrix} \tag{6}$$

which points toward the center of curvature and, as easily seen by inner product formation, is normal to t (primes denote differentation with respect to arc length).

The equilibrium thermal curvature k is expressed in terms of boom diameter d and expansion coefficient ϵ as

$$k = (\epsilon/d)T \tag{7}$$

whereas in Ref. 5, T has a magnitude equal to the diametric temperature differential (e.g., because of heat radiation J, with an absorptivity a, conductivity K, and thickness h).

$$|T| = [aJd^2/(4Kh)]\ |t \times \sigma| \tag{8}$$

and a direction normal to t, in the plane of t and σ, so that

$$T = (aJd^2/4Kh)\, t \times (t \times \sigma) \tag{9}$$

and therefore, with

$$C \triangleq (a\epsilon Jd/4Kh) \tag{10}$$

it follows that

$$k = -C\{\sigma - (\sigma \cdot t)t\} \tag{11}$$

From Eqs. (3, 5, and 11), with w' denoted by θ, the condition for static equilibrium is

$$\frac{d}{dr}\begin{bmatrix} \sqrt{1-\theta^2} \\ \theta \end{bmatrix} = -C\left\{\begin{bmatrix} \pm\sqrt{1-\beta^2} \\ -\beta \end{bmatrix} + \left(\beta\theta \mp \sqrt{(1-\theta^2)(1-\beta^2)}\right)\begin{bmatrix} \sqrt{1-\theta^2} \\ \theta \end{bmatrix}\right\} \tag{12}$$

which is meaningful only if both components of the vector differential equation are equivalent. Appendix A verifies that this is the case, and subsequently validates the solution,

$$\theta = \pm\sin\{2\arctan(e^{\pm Cr}\tan\phi/2) - \phi\},\quad 0 \le r \le \ell \tag{13}$$

where

$$\phi \triangleq \arccos\ \sigma_x \tag{14}$$

while in general, signs in Eq. (13) are positive for acute ϕ and negative for obtuse ϕ. If the projection of t on σ reversed at any point along the deformed boom, however, the solution would have to be divided into two parts, using both sign conventions. This could happen, for example, with ϕ near 90 deg in Fig. 1a. All of these solutions disallow shadowing of any boom segment.

Equation (13) agrees with all known special conditions (e.g., zero slope at the base; no deformation for vanishing C or ϕ; nonnegative θ with $\sigma_y \le 0$), while providing a physically meaningful behavior for bounded static deformations. It also represents a more thorough and more stable modeling than that normally used for thermal analysis; Appendix B shows that, with the combined effects of repeated simplification, equations just presented reduce to familiar forms commonly used in conventional thermal analysis.

Conventional thermal analysis often replaces Eqs. (1) and (2) by Eqs. (B1) and (B2), with an approximate substitution for Eq. (B3) in which the last term is ignored. Omission of that term sacrifices positive definiteness, thus compromising stability. Another stabilizing feature, often dropped from thermal analysis, is the axial component in Eqs. (5) and (6), arising from fixed arc length in Eq. (4). Although both of these features were included in the dynamic studies of Refs. 5 and 6, it is not clear what effect they would have in the presence of thermal lags. These prospects might well be considered in future stability analyses of the type in Refs. 1-4, since instability invites just the kind of motion that tends to invalidate "small deflection" assumptions.

Appendix A: Solution of Differential Equations

The lower half of Eq. (12) states that

$$d\theta/dr = C[\beta(1-\theta^2) \pm \theta\sqrt{(1-\beta^2)(1-\theta^2)}] \tag{A1}$$

while, from the upper half of the same relation,

$$\left(\frac{-\theta}{\sqrt{1-\theta^2}}\right)\frac{d\theta}{dr} = -C[\pm\sqrt{1-\beta^2} + \beta\theta\sqrt{1-\theta^2} \mp (1-\theta^2)\sqrt{1-\beta^2}] = -C\theta[\beta\sqrt{1-\theta^2} \pm \theta\sqrt{1-\beta^2}] \tag{A2}$$

When both sides of this expression are divided by the parenthetical factor on the left, Eq. (A1) immediately results.

The solution to be verified, Eq. (13), is of the form

$$\theta = \sin(Z-\phi) \tag{A3}$$

for the case shown in Fig. 1a, where Z is defined such that

$$\tan(Z/2) = e^{Cr}\tan(\phi/2) \tag{A4}$$

When these expressions are substituted into the derivative of Eq. (13)

$$\frac{d\theta}{dr} = C\cos(Z-\phi)\,\frac{2\tan(Z/2)}{1+\tan^2(Z/2)} \equiv C\sin Z\ \cos(Z-\phi) \tag{A5}$$

which agrees with the right of Eq. (A1) expressed in terms of Z;

$$C[\sin\phi\ \cos^2(Z-\phi) + \sin(Z-\phi)\ \cos(Z-\phi)\ \cos\phi] = C\sin Z\cos(Z-\phi) \tag{A6}$$

A smiilar procedure readily verifies the solution for Fig. 1b, with obtuse ϕ and negative signs in Eq. (13).

Appendix B: Reduction of Analysis to Conventional Formulation

The discussion accompanying Eqs. (3 and 4) notes that, for present purposes, a deflection w can be chosen as the generalized coordinate for Eq. (2). With this substitution, and with V_E expressed as the difference between a "total" strain energy V_S and thermally induced strain energy V_t, Eq. (2) becomes

$$(\mathrm{d}/\mathrm{d}t)\,(\delta T/\delta \dot{w}) - \delta(T - V_S)\,\delta w = \delta V_t/\delta w \tag{B1}$$

This equation is compatible with the relations,

$$V_S = \frac{1}{2} EI \int_0^\ell (p'')^2 \mathrm{d}r \tag{B2}$$

and

$$V_t = EI \int_0^\ell (p'' \cdot k - \tfrac{1}{2}k^2)\,\mathrm{d}r \tag{B3}$$

At this point, two approximations are introduced. The last term of Eq. (B3) will be dropped and, in Eq. (5), the arc length constraint will be ignored. In that case, Eq. (6) is replaced by

$$p'' \doteq \begin{bmatrix} 0 \\ w' \\ 0 \end{bmatrix} \tag{B4}$$

so that Eq. (B3) reduces to

$$V_t = EI \int_0^\ell k_y w''\,\mathrm{d}r \tag{B5}$$

where, from Eq. (11),

$$k_y = -C[\sigma_2 - (\sigma \cdot t)\,w'] \tag{B6}$$

When the inner product is formed with the arc length constraint in Eq. (5) again ignored,

$$(\sigma \cdot t) \doteq \sigma_1 - \sigma_2 w' \tag{B7}$$

and, if quadratic terms are omitted when this is combined with the preceding equation,

$$|k_y| \doteq C(\sigma_2 - \sigma_1 w') \tag{B8}$$

This is the magnitude of the steady-state thermal curvature in conventional analysis [e.g., used in Eq. (8) of Ref. 1, based on the equivalent expression in Ref. 3, or obtained by setting $\delta k/\delta t$ to zero in Ref. 4, Eq. (5)], and dynamic analyses customarily use Eq. (B1) with the approximation in (B5).

References

[1] Augusti, G., "Comment on 'Thermally Induced Vibration and Flutter of Flexible Booms,'" *Journal of Spacecraft and Rockets*, Vol. 8, Feb. 1971, pp. 202-204.

[2] Jordan, P.F., "Comment on 'Thermally Induced Vibration and Flutter of a Flexible Boom,'" *Journal of Spacecraft and Rockets*, Vol. 8, Feb. 1971, pp. 204-205.

[3] Etkin, B. and Hughes, P.C., "Explanation of the Anomalous Spin Behavior of Satellites with Long Flexible Antennae," *Journal of Spacecraft and Rockets*, Vol. 4, Sept. 1967, pp. 1139-1145.

[4] Yu, Y.Y., "Reply by Author to P.F. Jordan and G. Augusti and New Results of Two-Mode Approximation Based on a Rigorous Analysis of Thermal Bending Flutter of a Flexible Boom," *Journal of Spacecraft and Rockets*, Vol. 8, Feb. 1971, pp. 205-208.

[5] Farrell, J.L. and Newton, J.K., "Continuous and Discrete RAE Structural Models," *Journal of Spacecraft and Rockets*, Vol. 6, April 1969, pp. 414-423.

[6] Farrell, J.L., Newton, J.K., Miller, J.A., and Solomon, E.N., "Optimal Estimation of Rotation – Coupled Flexural Oscillations," *Journal of Spacecraft and Rockets*, Vol. 6, Nov. 1969, pp. 1290-1298.

Maximum Response of Missiles due to Inertial Asymmetry

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Nomenclature

c	$= L_p/I_x$
d	= diameter
I, I_x	= transversal and axial moments of inertia, respectively
i	$= \sqrt{-1}$
I_{xz}	= centrifugal moment of inertia
$K_{1,2}$	= initial amplitudes of nutational and processional modes
K_T	= trim mode
$L(p)$	$= L_p p$, damping moment in roll due to p
$M(\alpha)$	$= M_{\alpha_1}\alpha = \tfrac{1}{2}\rho V^2 Sd\, C_{m_{\alpha_1}}\alpha$, restoring moment due to α
$M(q)$	$= M_q q$, damping moment due to q
p	= roll rate
q	= complex angular yawing velocity
S	= reference area
t	= time
V	= velocity
X, Y, Z	= body axis system
α	= complex angle of attack
α_1	= angle of attack in pitch
δ_α	$= I_{xz}/(I_x - I)$
η	$= pI_x/2I$
λ	$= M_q/2I$
$\lambda_{1,2}$	$= (1 \pm \tau)$
$\phi_{1,2}$	$= \lambda_{1,2} + i\omega_{1,2}$
ϕ	= roll angle
ρ	= air density
τ	$= \eta/\omega$
ω_0	$= (-M_{\alpha_1}/I)^{1/2}$
ω	$= (\omega_0^2 + \eta^2)^{1/2}$
$\omega_{1,2}$	$= \eta(1 \pm 1/\tau)$
$(\dot{\ })$	= derivation with respect to time

Subscripts

R	= resonance
0	= at $t = 0$

I. Introduction

THE dynamic response of missiles caused by configurational or inertial asymmetry is analyzed in detail in Ref. 1, and major results are summarized in Ref. 2. It was found that for exponentially varying roll rates the maximum response occurs generally after the so-called "steady-state linear theory resonance" ($p_R = \omega_{1_R}$).

It also was found that, in the case of roll acceleration or deceleration, the maximum response associated with

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